Metal Forming – BSc 2023/24-1

Strains – Stresses

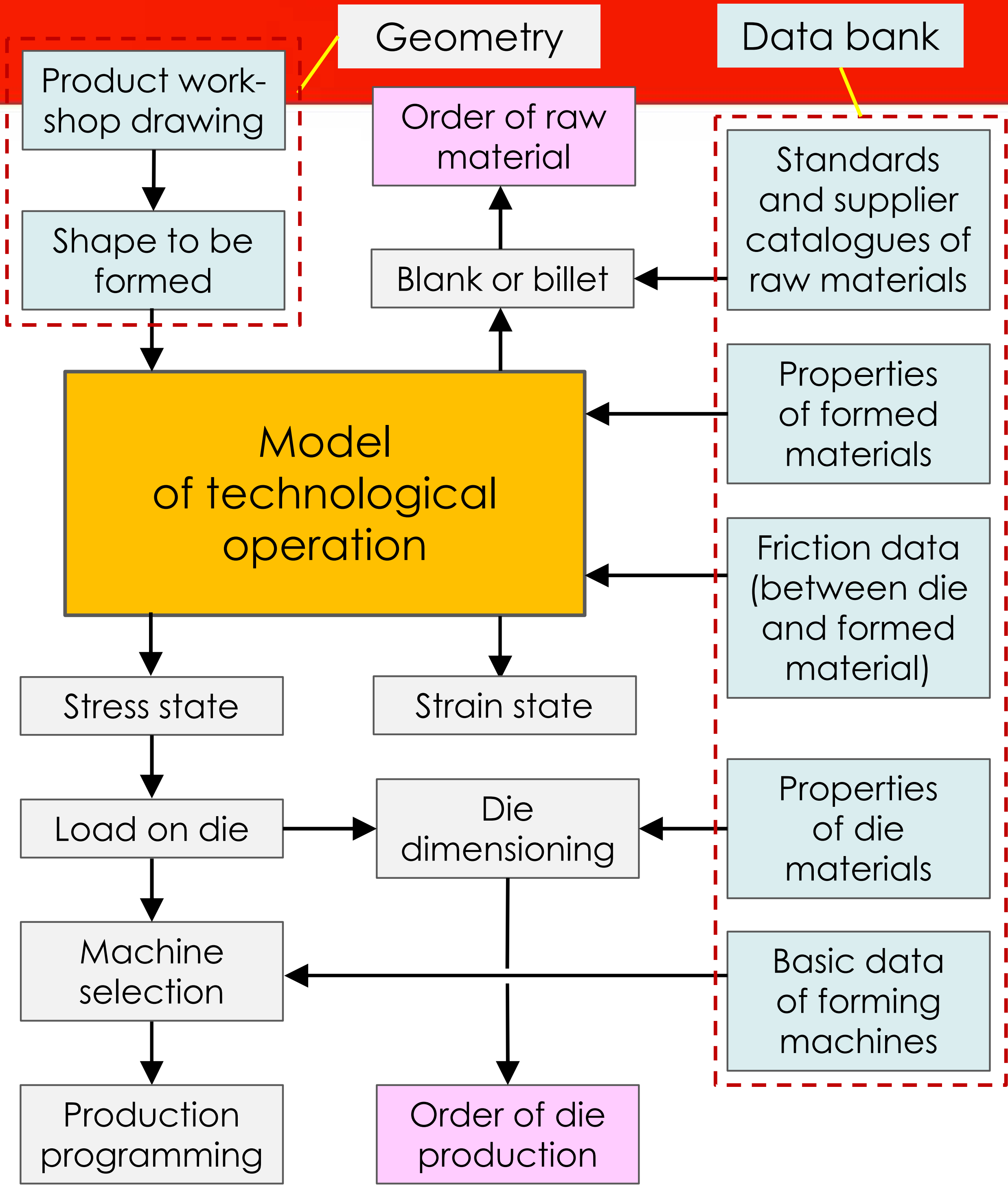
Goal of forming technologies:

Permanently change the **shape** of the initial part by using a die set, while the **properties** of the workpiece material are also changing.

The formed piece **resists** the deformation, which generates stresses/forces as an **answer** for the deformation constraints.

The designing of the forming technology **starts** from the **ready workpiece** back to the available **raw material**, through the proper technology steps.

Planning of a forming technology



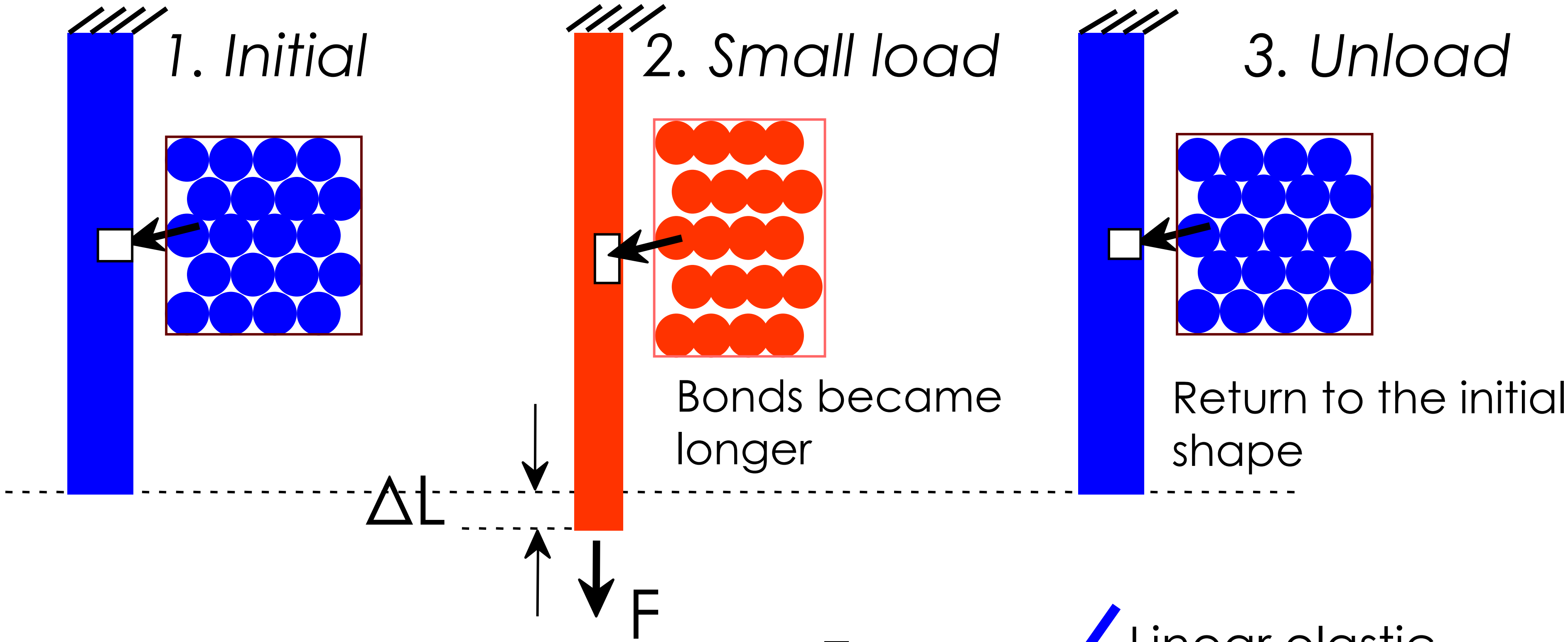
Deformation:

During forming operations **elastic** and **plastic** deformation happens.

The elastic part is **reversible**, and the plastic part is **irreversible**, it remains.

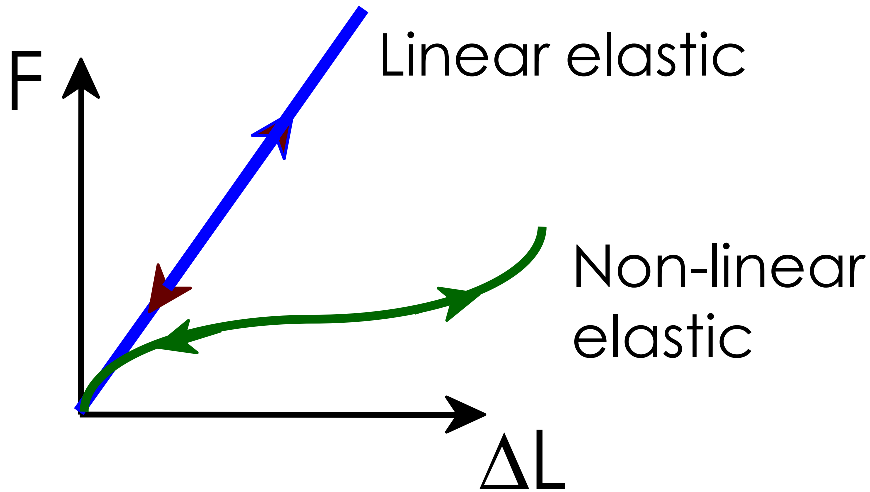
In several cases the elastic part is negligible.

Elastic deformation

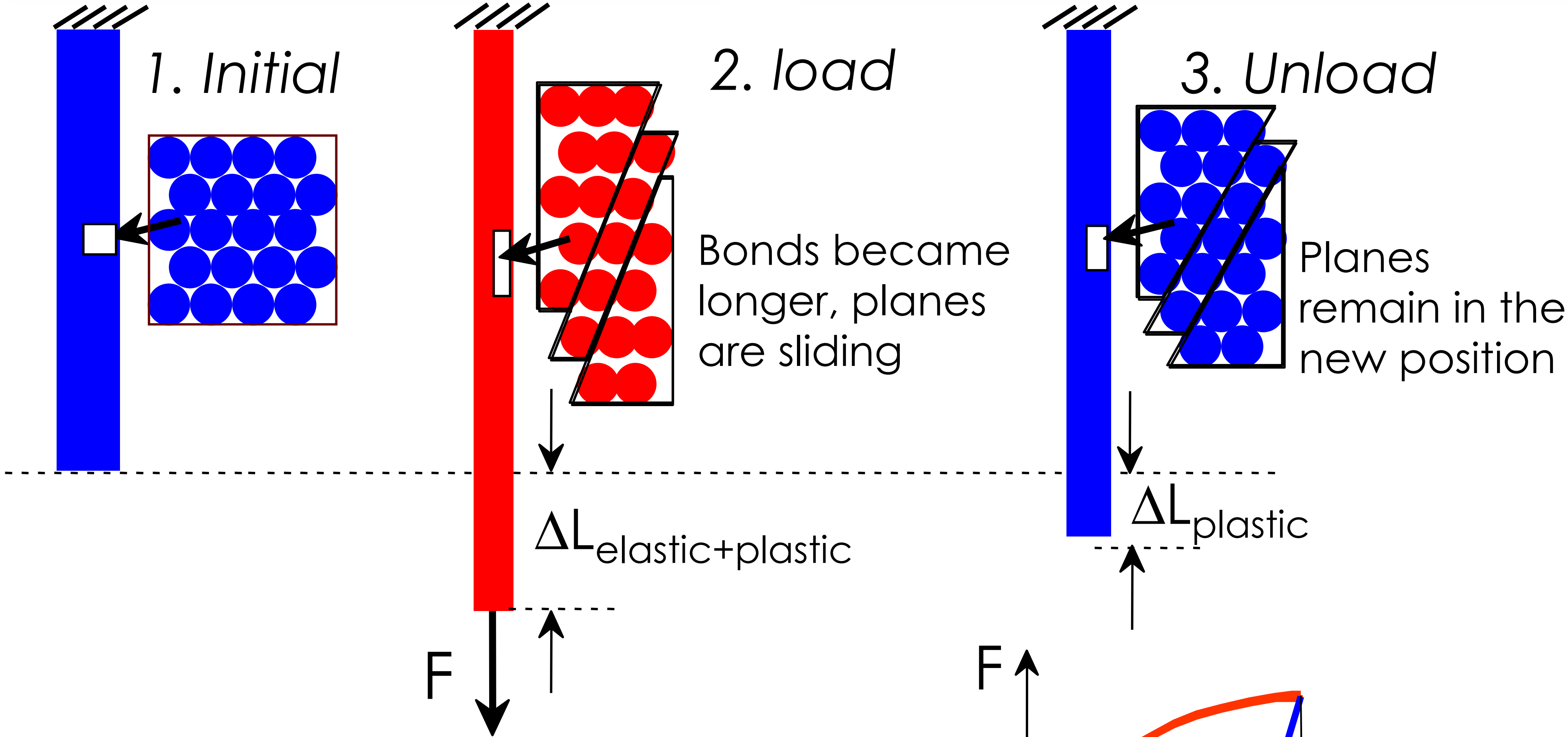


Elastic – reversible

The volume is not constant during elastic deformation.

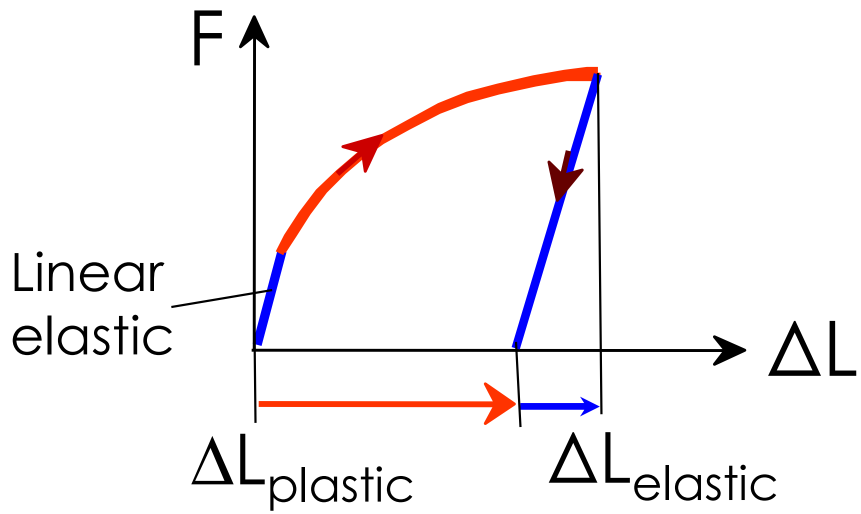


Plastic deformation



Plastic – remaining

The volume is constant during plastic deformation.



Tensile and compression

Strain

$$\varepsilon = \frac{l - l_0}{l_0}$$

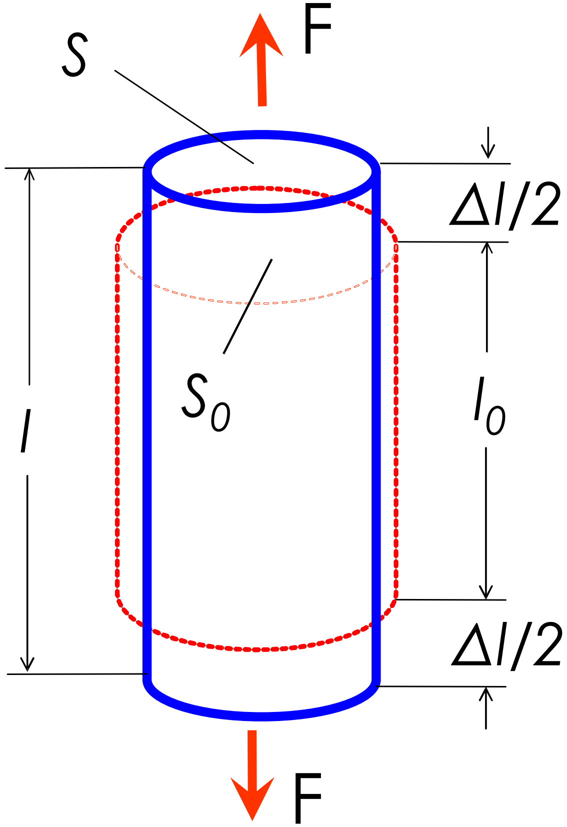
Stress

$$\sigma = \frac{F}{S} \approx \frac{F}{S_0}$$

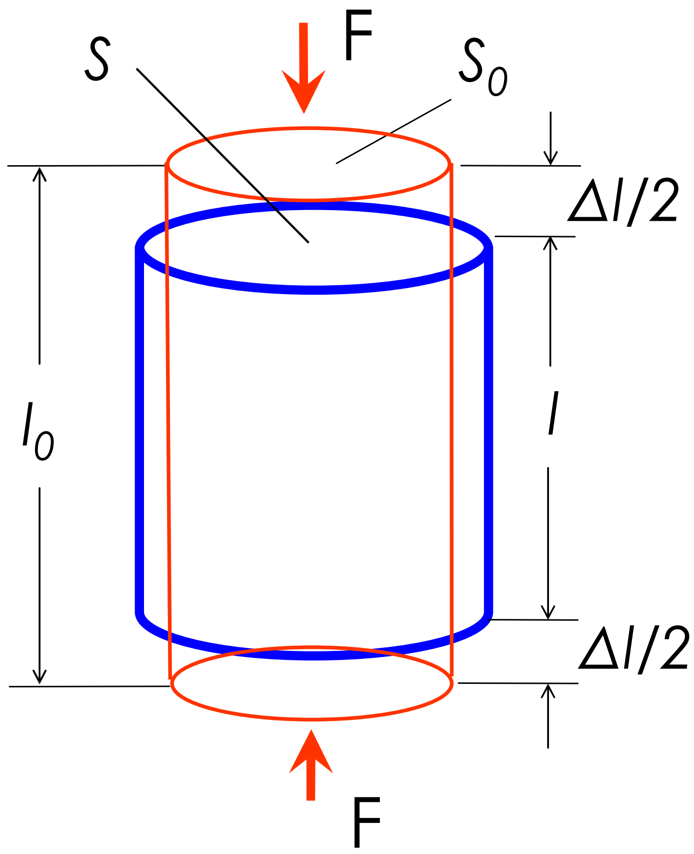
In elastic state

$$\sigma = E\,\varepsilon$$

(Hooke's law)

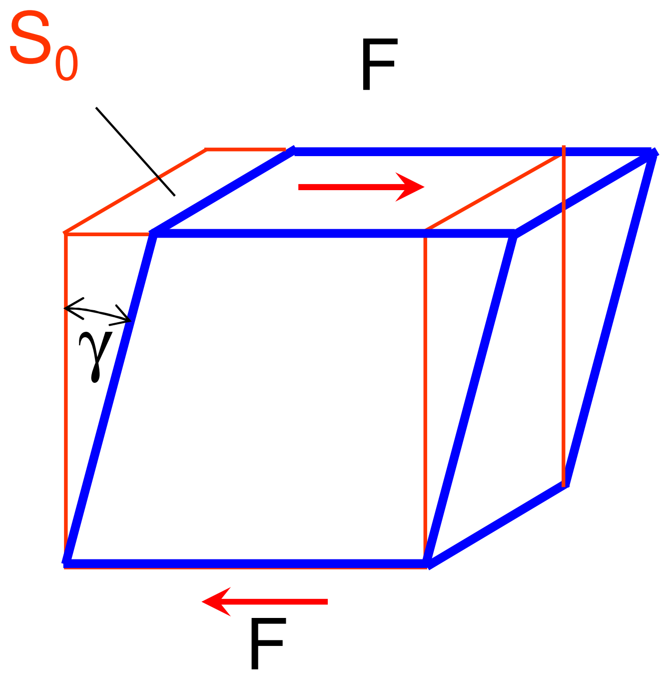


tension



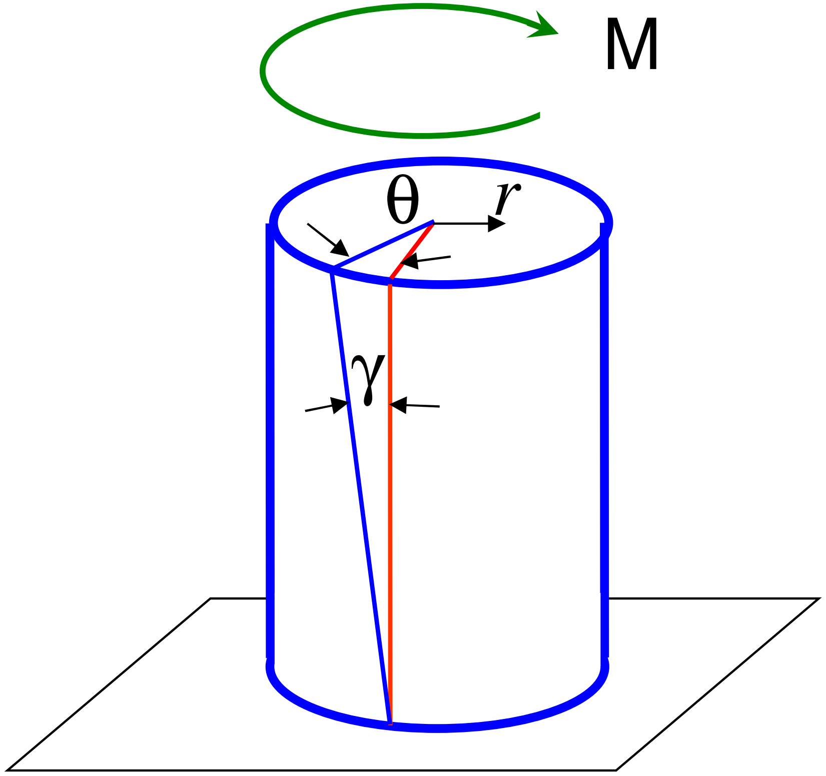
compression

Shear



Simple shear

$$\tau = \frac{F}{S} \approx \frac{F}{S_0}$$



Torsion

$$\tau = \frac{M}{I_p} r$$

In elastic state

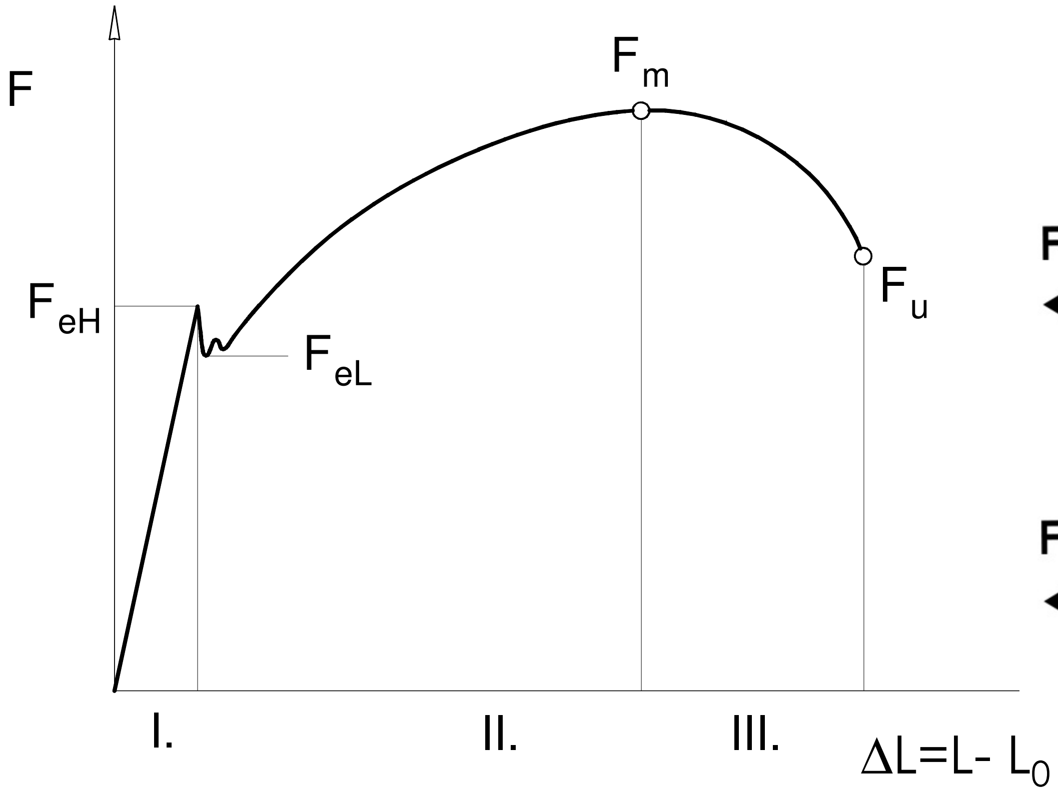
$$\tau = G\gamma$$

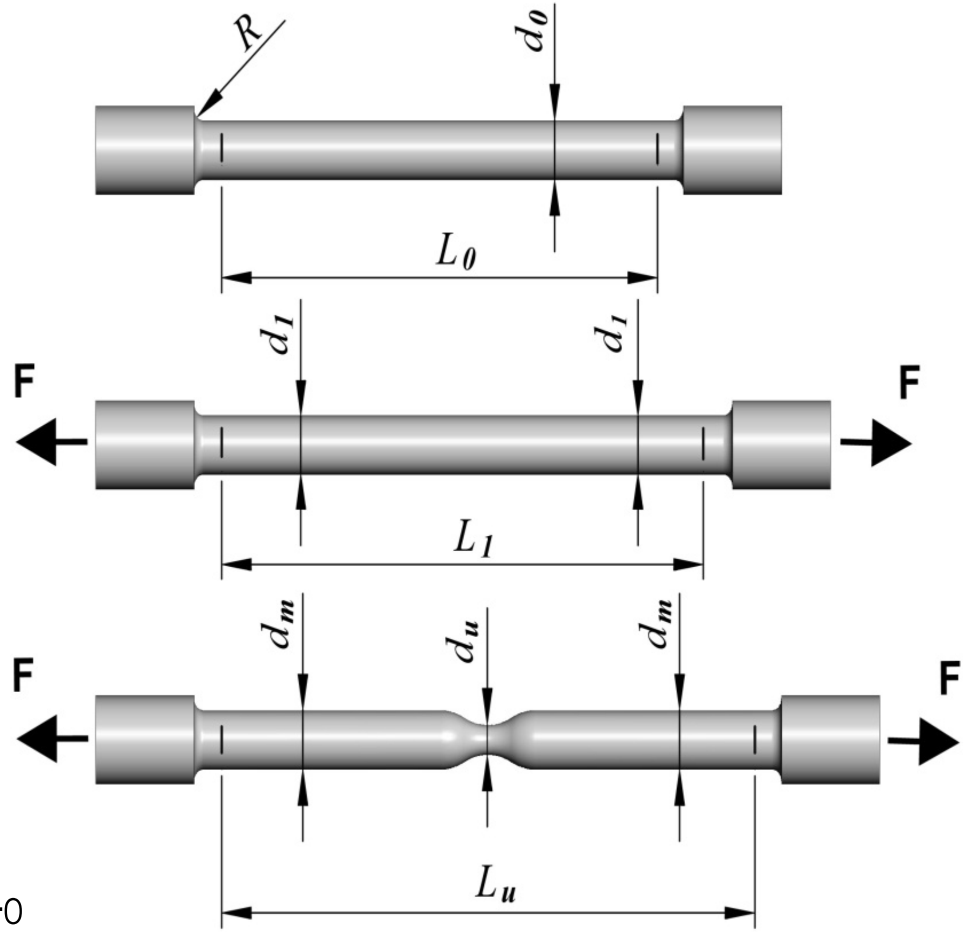
Mechanical properties

tensile, compression, torsion test

Tensile test

Tensile diagram





I. Elastic deformation
II. Uniform plastic deformation
III. Necking (non-uniform (localised) plastic deformation)

Standard tensile test results

Stress

Yield stress (MPa)

$$R_e = \frac{F_e}{S_0}$$

$$R_{eH} = \frac{F_{eH}}{S_0}, \qquad R_{eL} = \frac{F_{eL}}{S_0}$$

$$R_{p0,2} = \frac{F_{p0,2}}{S_0}$$

Tensile strength (MPa)

$$R_m = \frac{F_m}{S_0}$$

Deformation

Contraction

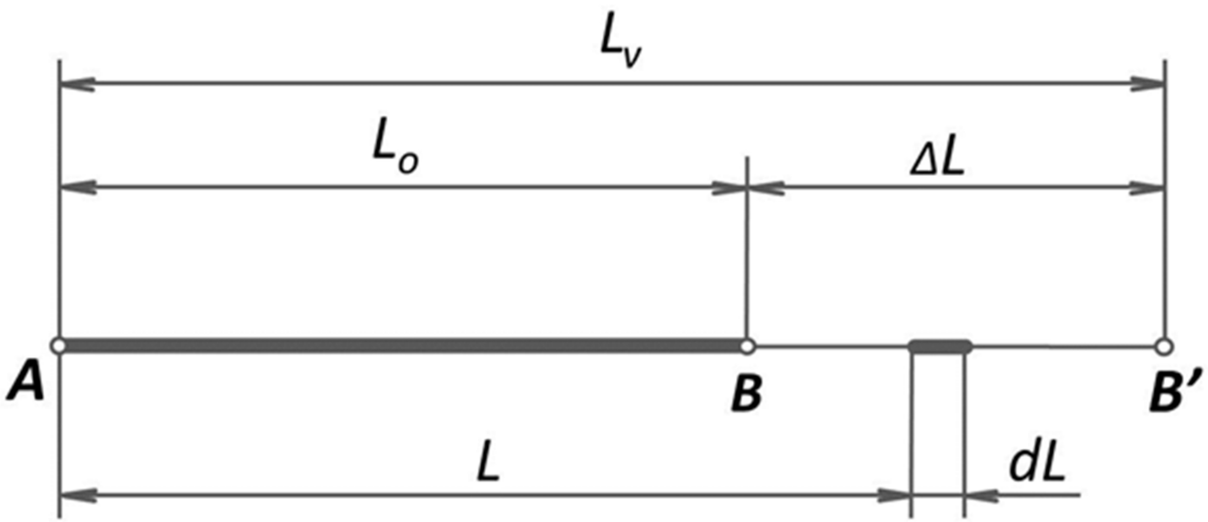
$$Z = \frac{S_0 - S_u}{S_0} 100 \ (\%)$$

Elongation

(engineering strain at fracture)

$$A = \frac{L_u - L_0}{L_0} 100 \ (\%)$$

Engineering & true strain



Engineering strain

$$d\varepsilon = \frac{dL}{L_0}$$

$$\varepsilon = \int_{L_0}^{L_v} \frac{dL}{L_0} = \frac{L_v - L_0}{L_0} = \frac{\Delta L}{L_0}$$

True strain

$$d\varphi = \frac{dL}{L}$$

$$\varphi = \int_{L_0}^{L_v} \frac{dL}{L} = \ln \frac{L_v}{L_0}$$

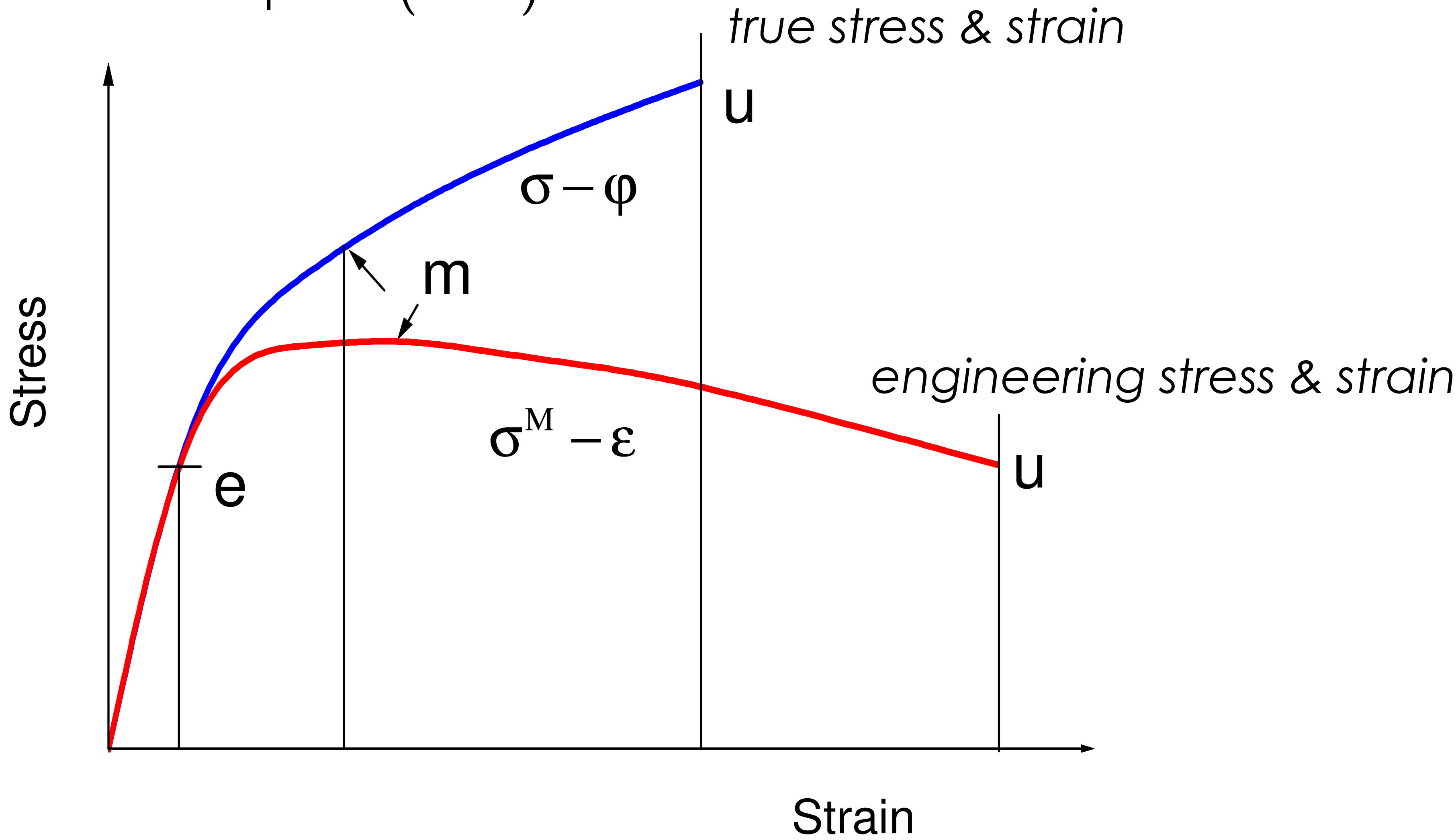
Engineering & true mechanical quantities

Engineering strain, stress		*True strain, stress*
$\varepsilon = \dfrac{l - l_0}{l_0}$ $\varepsilon = \dfrac{S_0}{S} - 1$	**Strain**	$\varphi = \ln \dfrac{l}{l_0}$ $\varphi = \ln \dfrac{S_0}{S}$
$\sigma^M = \dfrac{F}{S_0}$	**Stress**	$\sigma = \dfrac{F}{S}$
$W_c = \int_0^{\varepsilon_u} \sigma^M \, d\varepsilon$	**energy per unit volume (J/cm³)**	$W_c = \int_0^{\varphi_u} \sigma \, d\varphi$

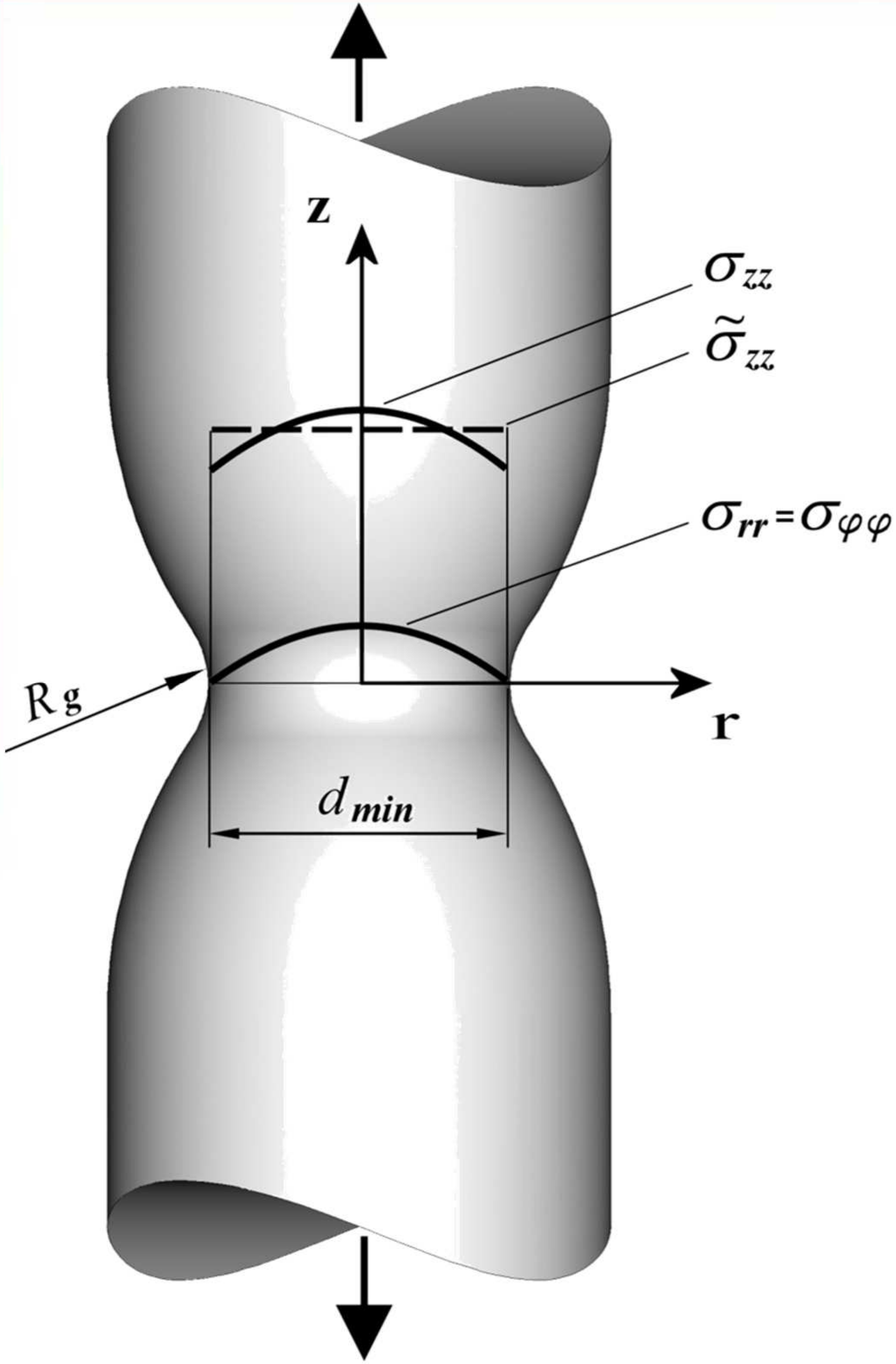
Stress-strain curves

$$F=\sigma S=\sigma^{M} S_0 \quad \Rightarrow \quad \sigma=\sigma^{M}\left(1+\varepsilon\right)$$

$$\varphi=\ln\left(1+\varepsilon\right)$$



Stress state at contraction



$$\sigma_{zz} = \bar{\sigma}\left[1+\ln\left(1+\frac{r_{\min}^2 - r^2}{2r_{\min}R_g}\right)\right]$$

$$\sigma_{rr} = \sigma_{\varphi\varphi} = \sigma_{zz} - \bar{\sigma}$$

$$\varphi_z = 2\ln\frac{d_0}{d_{\min}}$$

$$\varphi_r = \varphi_\varphi = \ln\frac{d_{\min}}{d_0}$$

$$\bar{\varphi} = \varphi_z$$

$\bar{\sigma}$ – *equivalent stress*

$\bar{\phi}$ – *equivalent strain*

Linear elastic properties

Hooke's law

$$\sigma = E\,\varepsilon$$

Elastic modulus:

E (Young-modulus)

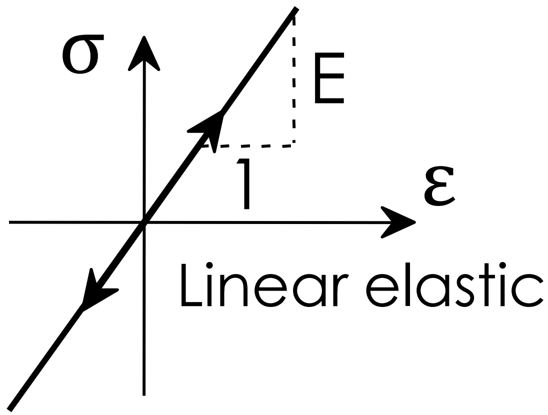
Poisson ratio, ν:

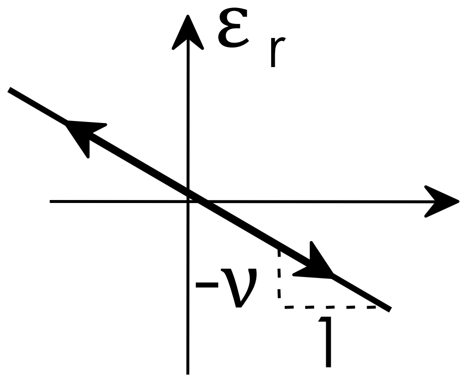
$$\nu = -\frac{\varepsilon_r}{\varepsilon}$$

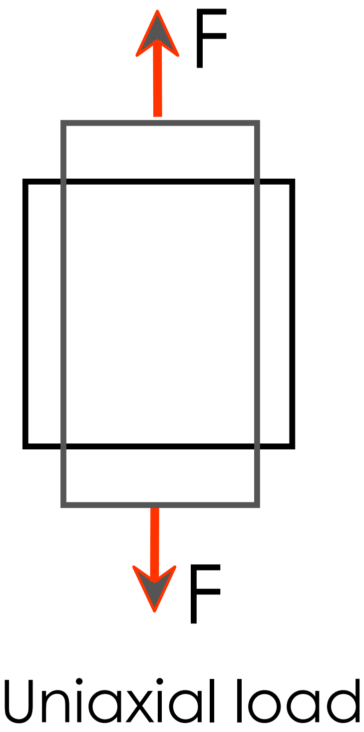
metals: $\nu \sim 0{,}33$
ceramics: $\nu \sim 0{,}25$
polymers: $\nu \sim 0{,}40$

units:

E: (GPa) or (MPa)
ν: no dimension







ε_r - radial strain

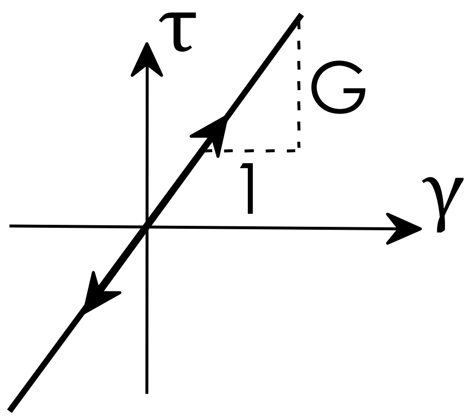
$$E_{ceramics} > E_{metal} >> E_{polymer}$$

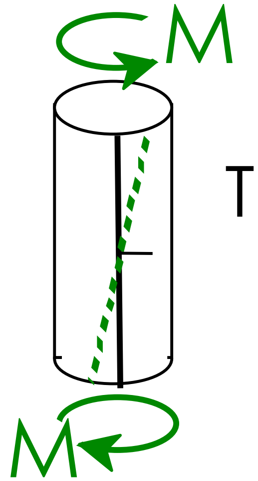
Linear elastic (shear) properties

Hooke's law

$$\tau = G\,\gamma$$

Shear modulus, G





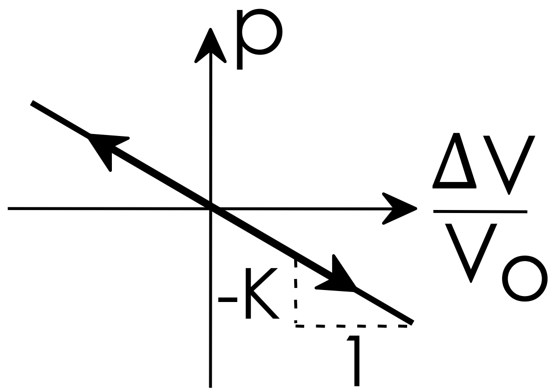
Torsion test

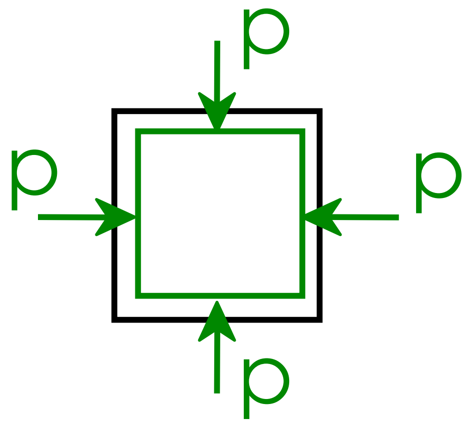
$$G = \frac{E}{2(1+\nu)}$$

Bulk modulus, K

$$p = -K\frac{\Delta V}{V_o}$$

$$K = \frac{E}{3(1-2\nu)}$$



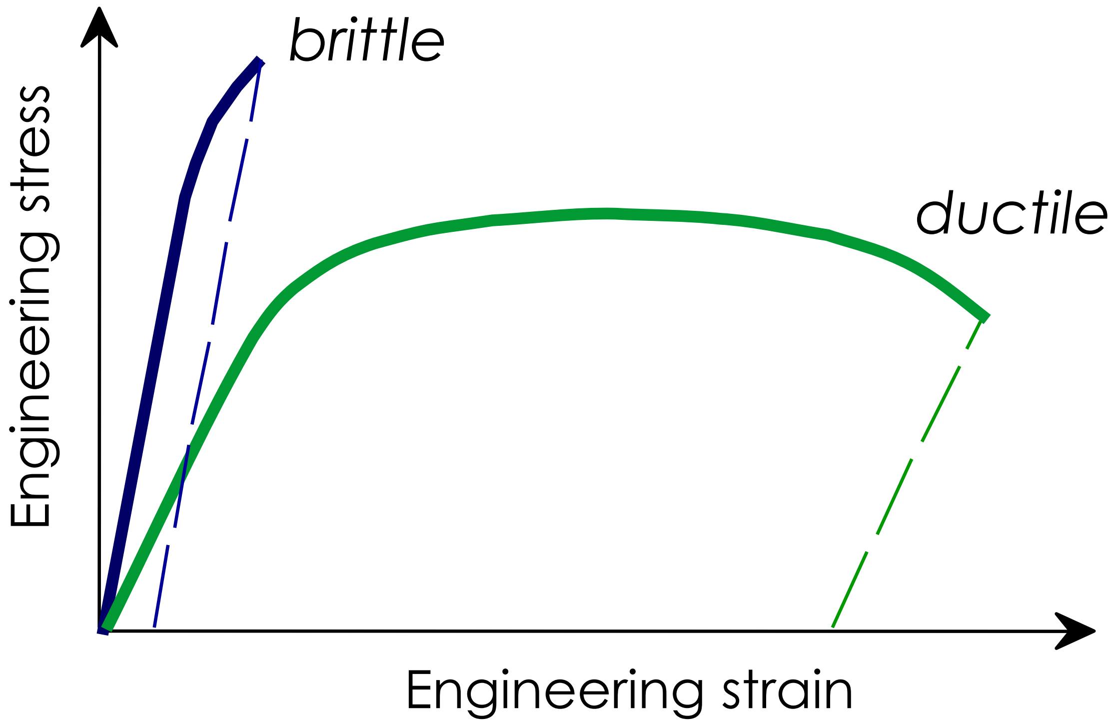


under hydrostatic pressure:

initial volume: V_o

volume change: ΔV

Ductile – brittle behavior

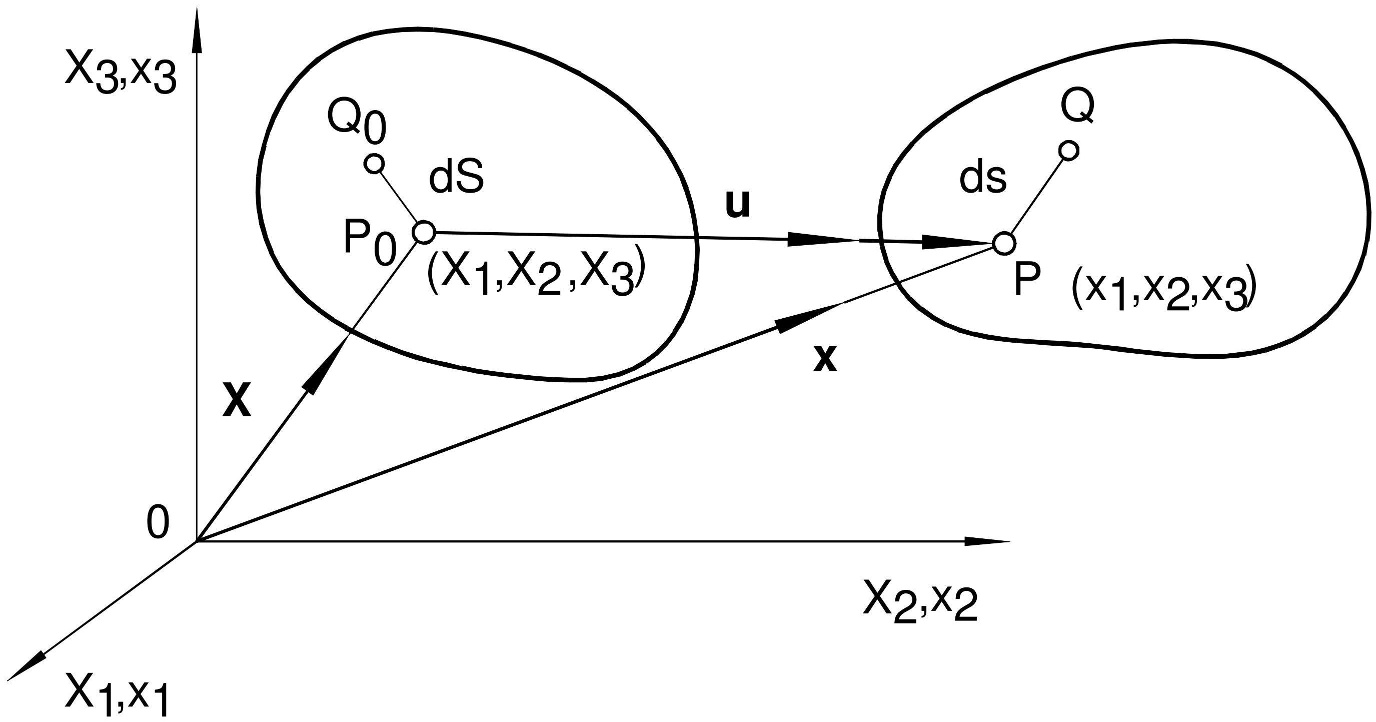


brittle - if the remaining (plastic) deformation ≈ 0
ductile - if the remaining (plastic) deformation is significant

Deformation - strain

continuum mechanical description

Motion of a body

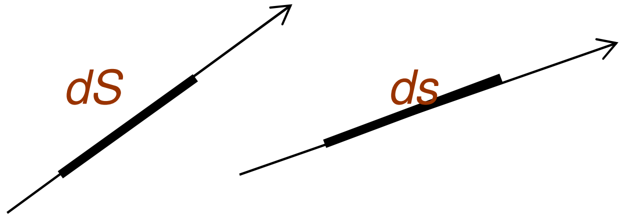


The motion of the body is described in a coordinate system. The points, lines and volume elements of the body are described in this system during the deformation.

Stretch ratio, engineering strain

Stretch ratio

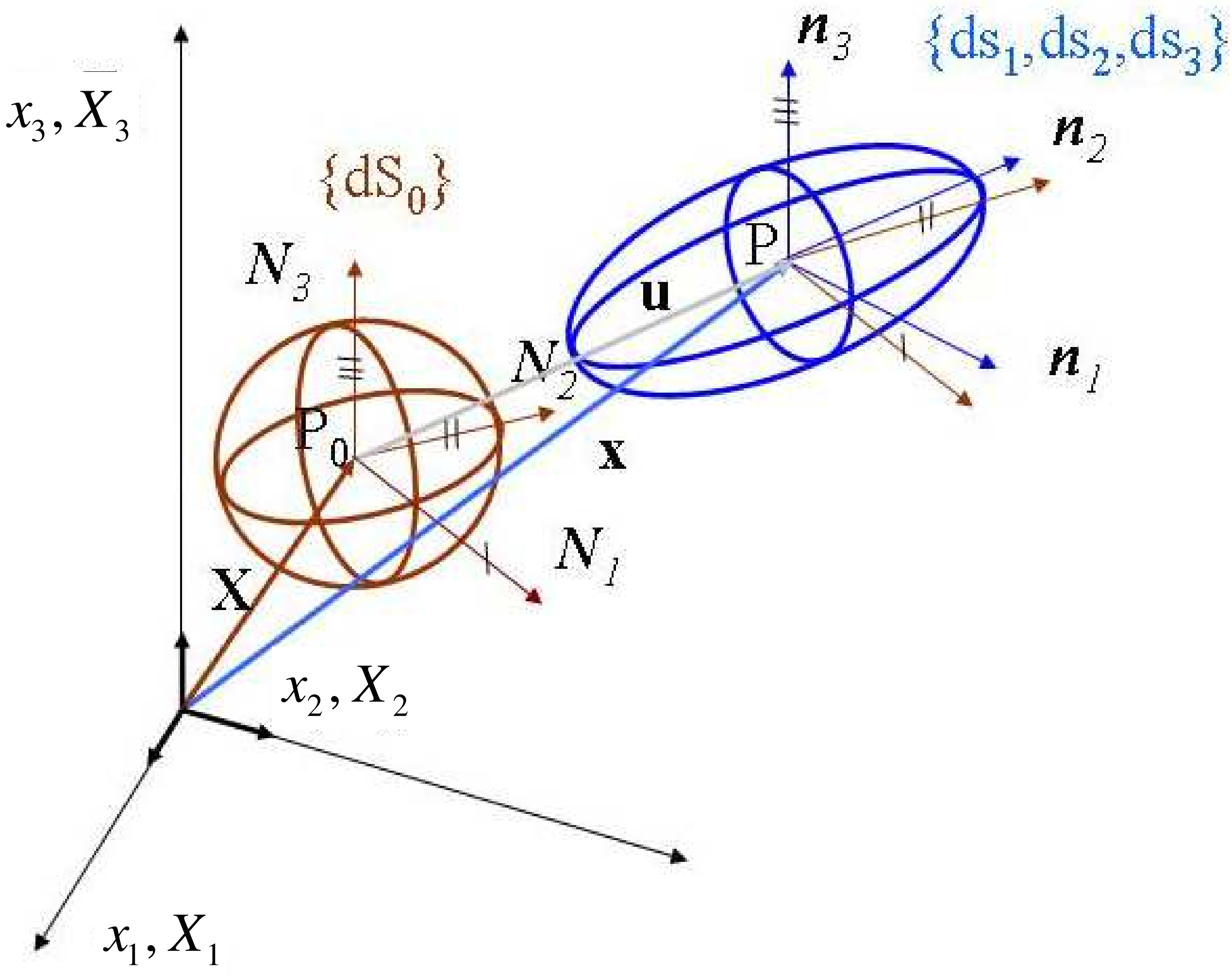
$$\lambda = \frac{ds}{dS}$$



Engineering strain

$$\varepsilon = \frac{ds - dS}{dS} = \frac{ds}{dS} - 1 = \lambda - 1$$

Logarithmic (true) strain



A (small) sphere in the environment of point P_0 at t=0, will be transformed to an ellipsoid during the deformation.

Logarithmic (true) strain

dS – sphere diameter, ds_i – axes of ellipsoid, $ds_1 > ds_2 > ds_3$

$$\varphi_1 = \ln\lambda_1 = \ln\frac{ds_1}{dS},\ \varphi_2 = \ln\lambda_2 = \ln\frac{ds_2}{dS},\ \varphi_3 = \ln\lambda_3 = \ln\frac{ds_3}{dS}$$

$$\boldsymbol{\varphi} = \begin{bmatrix} \varphi_1 & 0 & 0 \\ 0 & \varphi_2 & 0 \\ 0 & 0 & \varphi_3 \end{bmatrix}$$

Equivalent strain, stain rate

Tensor quantity characterized with a scalar value.

$$\bar{\varepsilon} = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_{11}-\varepsilon_{22})^2+(\varepsilon_{11}-\varepsilon_{33})^2+(\varepsilon_{22}-\varepsilon_{33})^2+6\left(\varepsilon_{12}^2+\varepsilon_{13}^2+\varepsilon_{23}^2\right)}$$

This **equivalent strain** is used to compare different state of strains.

Strain rate

From velocity field: $\xi_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$

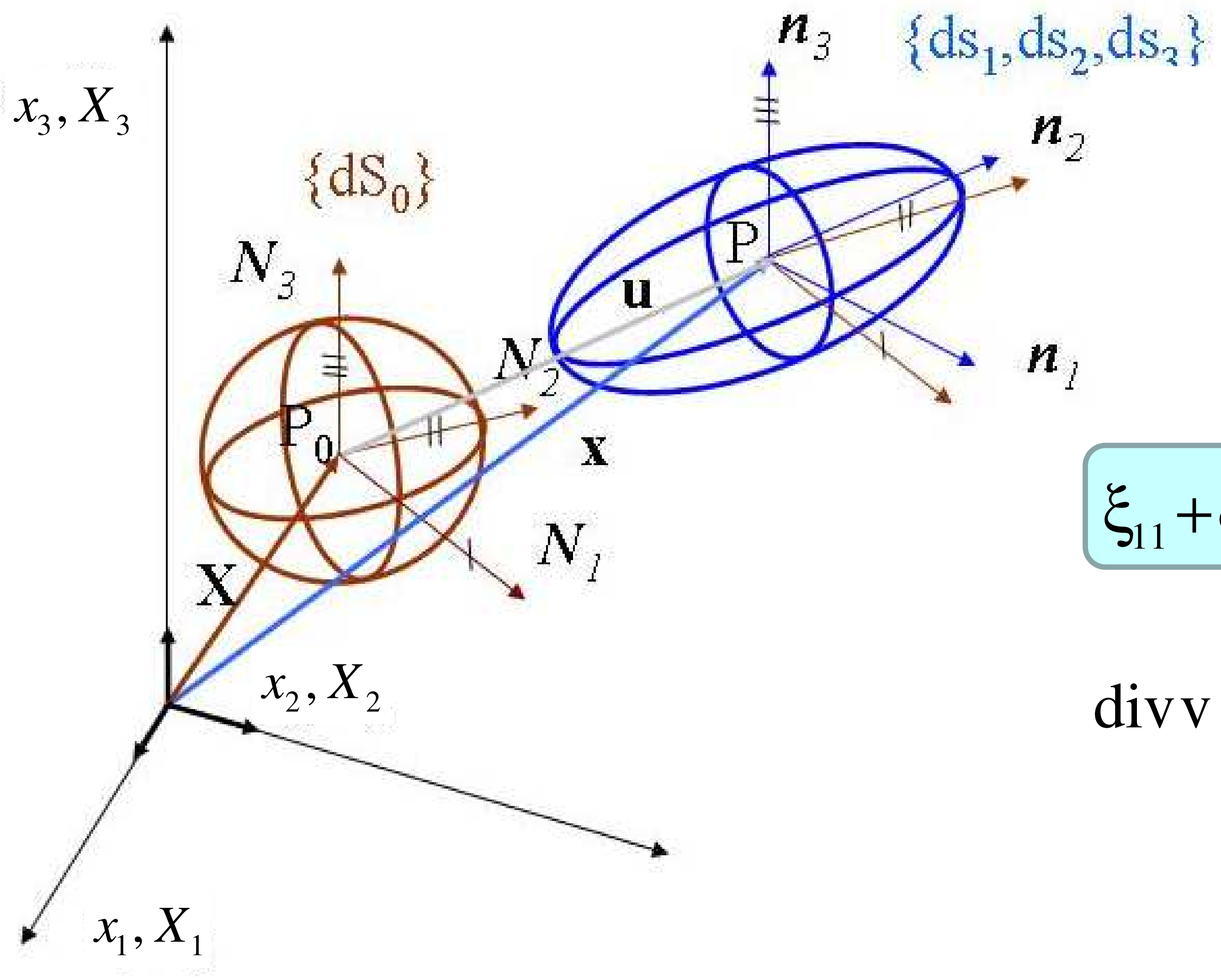
Equivalent strain rate:

$$\bar{\xi} = \frac{\sqrt{2}}{3}\sqrt{(\xi_{11}-\xi_{22})^2+(\xi_{11}-\xi_{33})^2+(\xi_{22}-\xi_{33})^2+6\left(\xi_{12}^2+\xi_{13}^2+\xi_{23}^2\right)}$$

Equivalent strain: $\bar{\varepsilon} = \int_{t_0}^{t} \bar{\xi}\,dt$

Volume constancy

$$dS_1 dS_2 dS_3 = ds_1 ds_2 ds_3 \quad \Rightarrow \boxed{\varphi_1 + \varphi_2 + \varphi_3 = 0}$$



$$\boxed{\xi_{11} + \xi_{22} + \xi_{33} = 0}$$

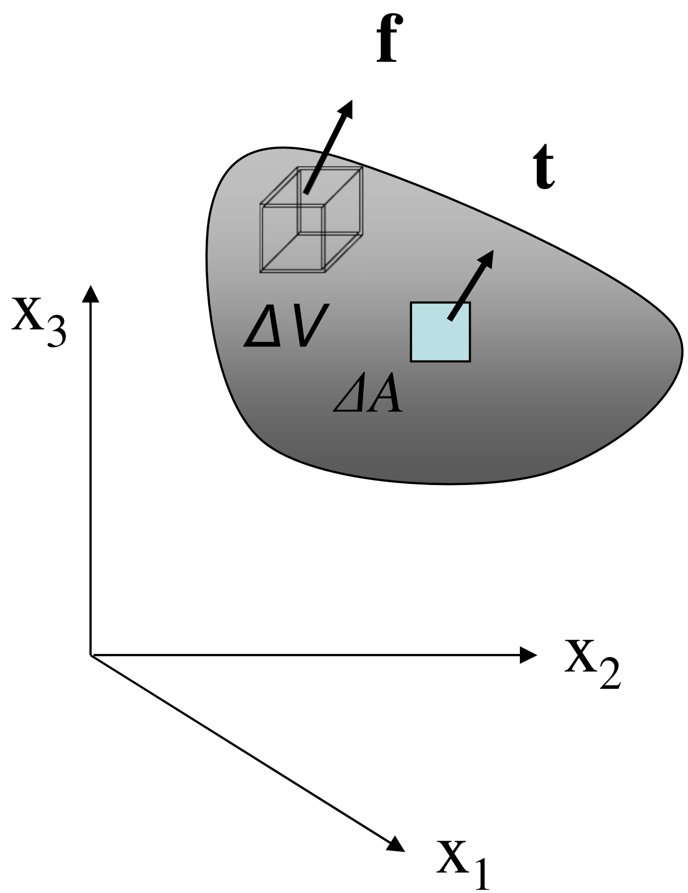
$$\operatorname{div} \mathrm{v} = 0,$$

Stress

continuum mechanical description

Volume and surface forces

External forces act on a body with V_0 volume and A_0 surface, therefor it undergoes deformation; Volume and surface changes to V and A respectively. The external forces can be volume and surface forces.



Surface force density

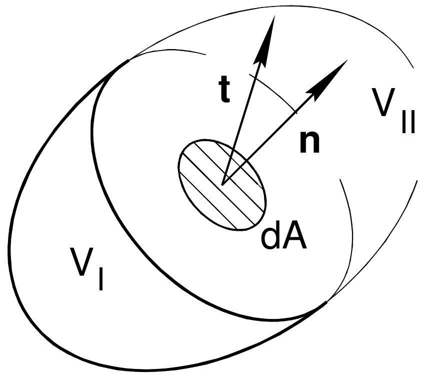
$$\mathbf{t} = \lim_{\Delta A \to 0} \frac{\Delta \mathbf{F}}{\Delta A}$$

Volume force density

$$\mathbf{f} = \frac{1}{\rho} \lim_{\Delta V \to 0} \frac{\Delta \mathbf{F}}{\Delta V}$$

Volume forces: weight, magnetic forces

Stress tensor



Cut the body into two and apply surface forces on the cut surface to keep on the equilibrium.

$$t_i = \sigma_{ij} n_j, \qquad \mathbf{t} = \boldsymbol{\sigma}^T \cdot \mathbf{n}$$

$$t_1 = \sigma_{11} n_1 + \sigma_{12} n_2 + \sigma_{13} n_3$$

$$t_2 = \sigma_{21} n_1 + \sigma_{22} n_2 + \sigma_{23} n_3$$

$$t_3 = \sigma_{31} n_1 + \sigma_{32} n_2 + \sigma_{33} n_3$$

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

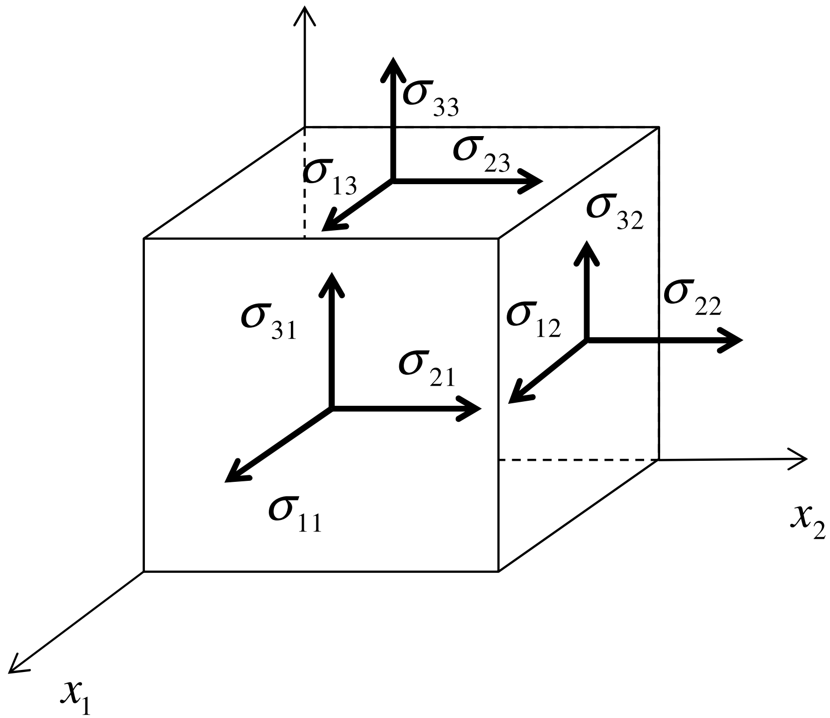
$\boldsymbol{\sigma}$ – Cauchy stress tensor

Stress tensor

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Diagonal elements:	normal stresses
Off-diagonal elements:	shear stresses

Normal stress:	positive if tensile
	negative if compressive



Equivalent stress

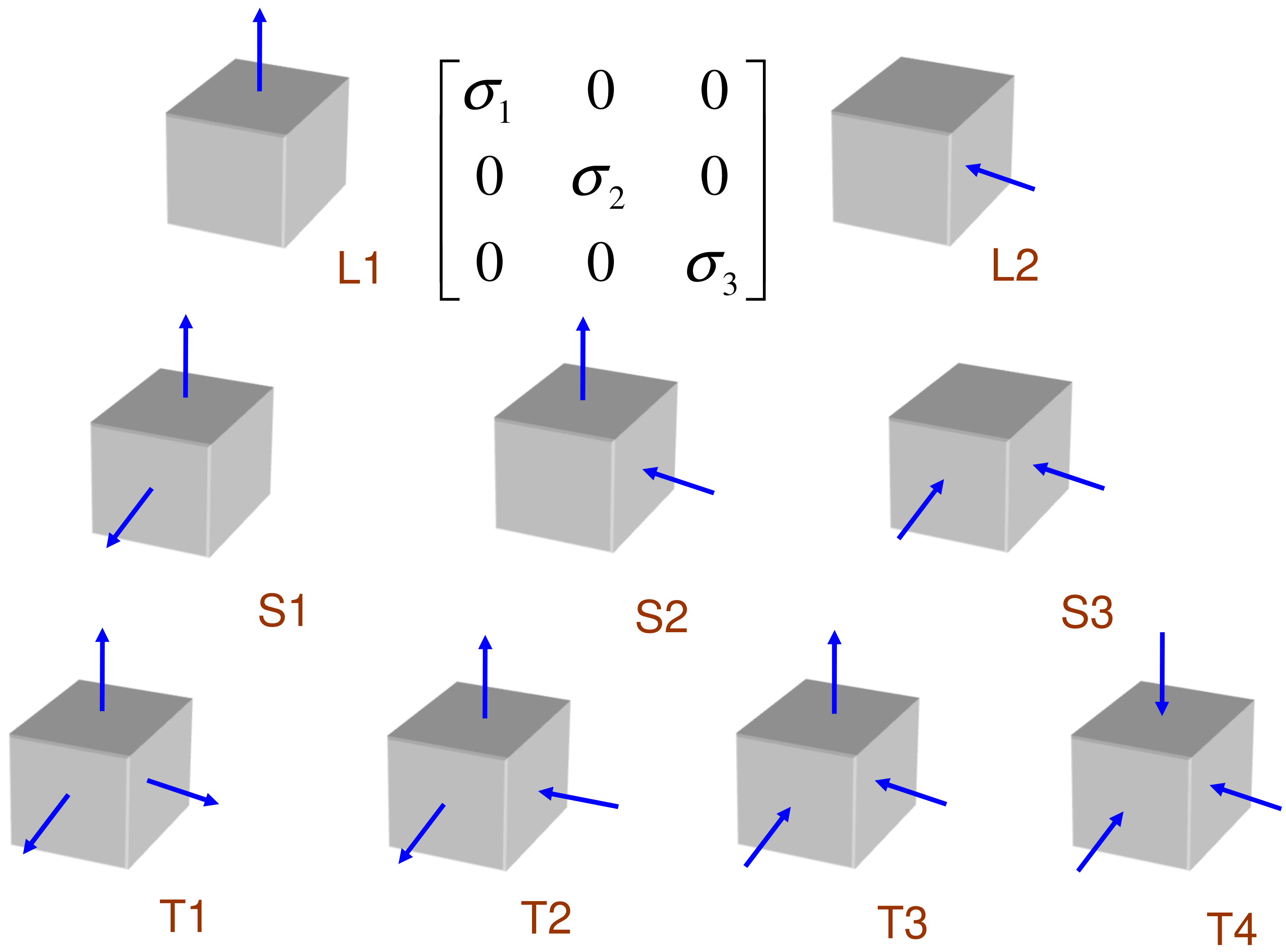
Equivalent stress – by von Mises or Huber-Mises-Hencky theory

The tensor quantity characterized with a scalar value:

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2 + 6\left(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2\right)}$$

The equivalent stress is used to compare different state of stresses.

Stress states



$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Constitutive law

Stress - strain relation

Plastic behavior of the material

von Mises yield criterion:
(or Huber-Mises-Hencky)

$$f = \sigma_{mises} - \sigma_{flow} = 0$$

$$\sigma_{mises} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{33} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + 6(\sigma_{12}^2 + \sigma_{12}^2 + \sigma_{12}^2)}$$

σ_{flow} – flow stress, material property, measured value

The flow stress is needed to start then maintain the plastic deformation in uniaxial state of stress.

σ_{flow} = F (eq. strain, eq. strain rate, temperature)

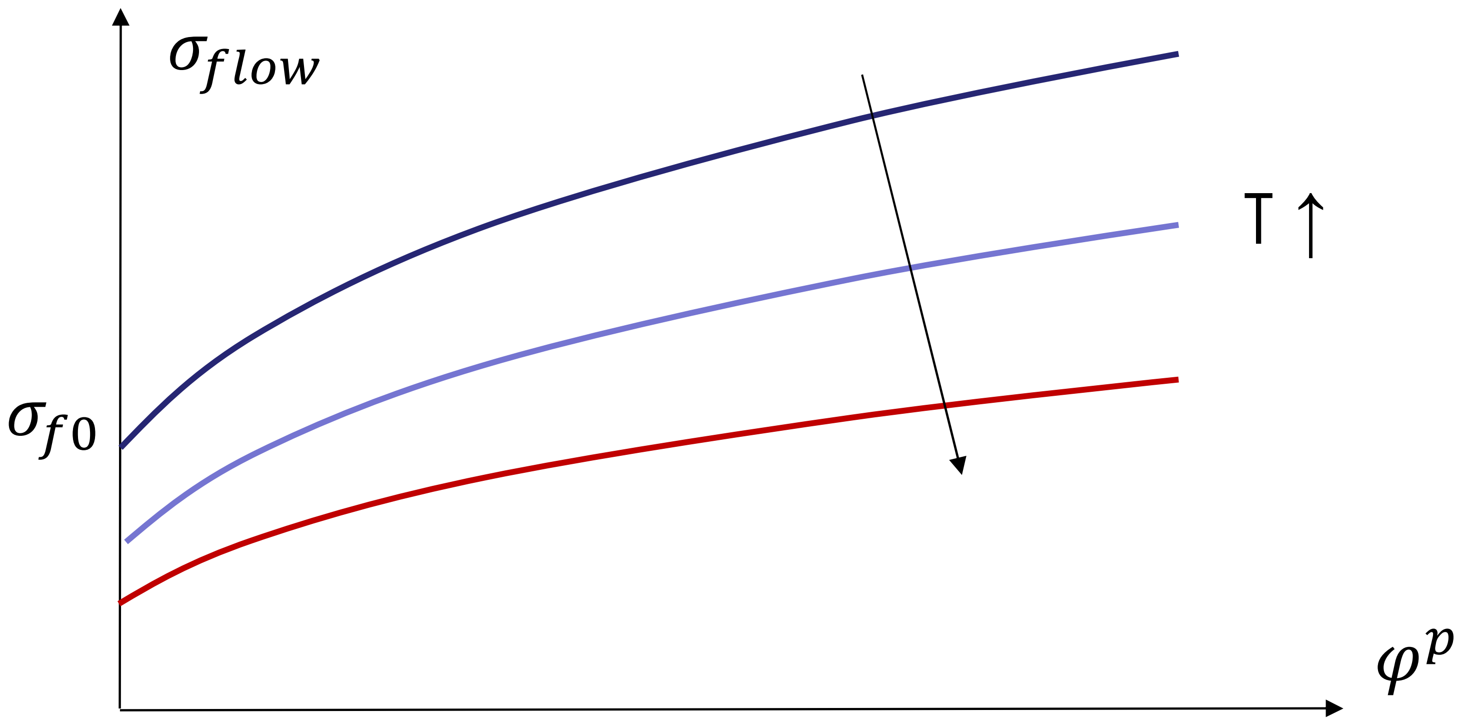
Condition of plastic flow:

$f < 0$	elastic state
$f = 0$	plastic deformation – flow
$\cancel{f > 0}$	physically does not exist

Plastic behavior of the material

Hardening

$$\sigma_{flow} = \sigma_{flow}(\varphi^p, \dot{\varphi}^p, T)$$



Plastic behavior of the material

Flow curves

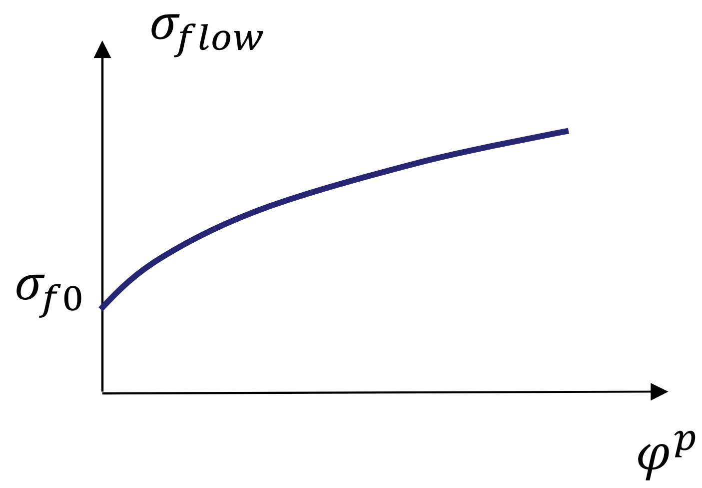
for cold forming

$$\sigma_f = C\,\bar{\varphi}^n$$

$$\sigma_f = C_1 + C_2\,\bar{\varphi}^n$$

$$\sigma_f = C_1 + C_2\,\bar{\varphi}^n - C_3\,e^{-C_4\bar{\varphi}}$$

$$\sigma_f = C_0 + C_1\left(1-\exp\left(-C_2\bar{\varphi}\right)\right) + C_3\left(1-\exp\left(-C_4\bar{\varphi}\right)\right) + C_5\left(1-\exp\left(-C_6\bar{\varphi}\right)\right)$$



for hot forming

$$\sigma_f = C\,\dot{\bar{\varphi}}^m$$

$$\sigma_f = \sigma_{f0}\,\bar{\varphi}^n \left(\frac{\dot{\bar{\varphi}}}{\dot{\bar{\varphi}}_0}\right)^m \exp\left(-\beta\,\Delta T\right)$$